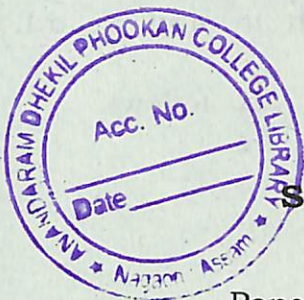


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1 (Sem-5/FYUGP) STA01MJ

2025

**STATISTICS**

(Major)

Paper : STA0500104

**(Sampling Distributions and  
Test of Significance)**

Full Marks : 60

Time : 2½ hours

***The figures in the margin indicate  
full marks for the questions.***

1. Answer the following questions as directed :  
1×8=8

(a) S.E. of a statistic may be reduced by increasing the sample size.

(State True **or** False)

(b) The mean of the chi-square distribution is \_\_\_\_\_ of its variance.

(Fill in the blank)

- (c) If  $X_1$  and  $X_2$  are two independent chi-square variates with  $n_1$  and  $n_2$  d.f. respectively then  $U = \frac{X_1}{X_2}$  follows

(i)  $\beta_2 \left( \frac{n_1}{2}, \frac{n_2}{2} \right)$

(ii)  $\beta_1(n_1, n_2)$

(iii)  $F$ -distribution

(iv) None of the above

(Choose the correct option)

- (d) Student's  $t$ -distribution curve is symmetrical about mean, it means that:

(i) Odd order moments are zero

(ii) Even order moments are zero

(iii) Both (i) and (ii)

(iv) None of (i) and (ii)

(Choose the correct option)

- (e) Karl Pearson's coefficient of skewness of the chi-square distribution with  $n$  d.f. is \_\_\_\_\_. (Fill in the blank)

- (f) Moment generating function of  $t$ -distribution does not exist.

(State True or False)

- (g) The sampling distribution of  $F$ -statistic does not involve any population parameters and depends only on the degrees of freedom.

(State True or False)

- (h) The 95% confidence limits for the population mean for large sample is \_\_\_\_\_. (Fill in the blank)

2. Answer **any six** from the following questions :  $2 \times 6 = 12$

(a) Define sampling distribution and standard error.

(b) Find the S.E. of sample mean.

(c) State *four* applications of chi-square distribution.

(d) Define non-central chi-square distribution.

(e) Find the M.G.F. of chi-square distribution.

(f) Define Student's  $t$ -distribution.

(g) Write *two* assumptions for student's- $t$  distribution. Also write the 99% confidence limits for population mean.

(h) If  $F(n_1, n_2)$  represents an  $F$ -variate with  $n_1$  and  $n_2$  d.f., prove that  $F(n_2, n_1)$  is distributed as  $1/F(n_1, n_2)$  variate.

(i) Define non-central  $F$ -distribution.

(j) Show that sample proportion is an unbiased estimate of the population proportion.

3. Answer **any four** from the following questions : 5×4=20

(a) Define random sample, parameter and statistic. Write *two* uses of standard error. 1+1+1+2=5

(b) Derive chi-square distribution.

(c) If  $X$  is chi-square variate with  $n$  d.f., then prove that for large  $n$ ,  
 $\sqrt{2X} \sim N(\sqrt{2n}, 1)$ .

(d) Prove that for large d.f.,  $t$ -distribution tends to standard normal distribution.

(e) If  $\rho = 0$ , then prove that

$$t = \frac{r}{\sqrt{1-r^2}} \cdot \sqrt{n-2} \text{ is distributed as}$$

student's  $t$  with  $(n-2)$  d.f.; where  $r$  is the sample correlation coefficient and  $\rho$  is the population correlation coefficient.

(f) Discuss the applications of  $F$ -distribution.



(g) When  $\nu_1 = 2$ , show that the significance level of  $F$  corresponding to a significant probability  $p$  is :  $F = \frac{\nu_2}{2} [p^{-(2/\nu_2)} - 1]$  where  $\nu_1$  and  $\nu_2$  have their usual meanings.

(h) Explain in brief the test used for testing the difference between two means for large samples.

4. Answer **any two** from the following questions :  $10 \times 2 = 20$

(a) (i) State and prove additive property of chi-square variate. 5

(ii) Derive cumulant generating function (c.g.f.) of chi-square distribution. Also find its mean and variance using c.g.f. 5

(b) Define non-central  $t$ -distribution and derive its probability density function.

(c) Define  $F$ -statistics and derive its distribution.

(d) Derive the relation between  $F$  and  $\chi^2$ -distribution.

(e) (i) Show that for  $t$ -distribution with  $n$  d.f., mean deviation about mean is given by :

$$\sqrt{n} \Gamma[(n-1)/2] / \sqrt{\pi} \Gamma(n/2)$$

5

(ii) In a sample of 1,000 people in Maharashtra, 540 are rice eaters and the rest are wheat eaters. Can we assume that both rice and wheat are equally popular in this state at 1% level of significance?

5

