1 (Sem-5/FYUGP) MAT 43 MJ

2025

MATHEMATICS

(Major)

Paper : MAT0500304

(Numerical Analysis—I)

Full Marks : 45

Time : 2 hours

The figures in the margin indicate full marks for the questions.

1. Answer the following as directed : $1 \times 5 = 5$

   (a) In the Gaussian elimination method, the coefficient matrix of the linear system of equations reduces to

   (i) upper triangular matrix

   (ii) diagonal matrix

   (iii) lower triangular matrix

   (iv) None of the above

   (Choose the correct option)

(b) Which of the following is an iterative method?

(i) Gauss-Jordan method

(ii) Lagrange's method

(iii) Gauss-Seidel method

(iv) Newton's divided difference method

(Choose the correct option)

(c) Evaluate $\Delta^2(3e^x)$.

(d) What is numerical integration? Mention one method of numerical integration.

(e) In Simpson's $\frac{1}{3}$rd rule, the degree of interpolating polynomial is

(i) 1

(ii) 3

(iii) 2

(iv) 5 (Choose the correct option)

2. Answer any *five* of the following questions : 2×5=10

(a) Prove the operator relation

$$(1+\Delta)(1-\nabla) \equiv 1$$

(b) Given $u_0 = 580$, $u_1 = 556$, $u_2 = 520$, $u_4 = 385$, find the value of u_3.

(c) A second-degree polynomial passes through (0, 1), (1, 3), (2, 7) and (3, 13). Find the polynomial.

(d) Construct a divided difference table for the following data :

x :	0	2	3	4	7	9
$f(x)$:	4	26	58	112	466	922

(e) Consider the following system of equations :

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

To use Jacobi iteration method

$$x^{(k+1)} = Hx^{(k)} + c$$

for solving this system, find out the coefficient matrix H.

(f) Write a short note on piecewise linear interpolation method.

(g) What is Richardson's extrapolation? Write briefly.

(h) Write the formulae for first- and second-order derivatives of $y = f(x)$ using Newton's forward difference interpolation formula.

(i) Write the general scheme of modified Euler's method.

(j) If $M(h) = \frac{1}{2h}[-y(x+2h) + 4y(x+h) - 3y(x)]$, show that

$$y'(x) - M(h) = c_1 h^2 + c_2 h^3 + c_3 h^4 + \ldots$$

where c_1, c_2, c_3, ... are constants independent of h.

3. Answer any *four* of the following questions : 5×4=20

(a) Find the number of students from the following data who secured marks not more than 45 :

Marks :	30–40	40–50	50–60	60–70	70–80
No. of students :	35	48	70	40	22

(b) Show that the nth divided difference of a polynomial of degree n is constant.

(c) Solve the following system of equations using Gauss elimination method :

$$2x_1 + 4x_2 + x_3 = 3$$
$$3x_1 + 2x_2 - 2x_3 = 2$$
$$x_1 - x_2 + x_3 = 6$$

(d) Construct the Gauss-Seidel scheme for the system

$$2x_1 - x_2 = 7$$
$$-x_1 + 2x_2 - x_3 = 1$$
$$-x_2 + 2x_3 = 1$$

(e) Find $f'(4)$ from the following table :

x :	1	2	4	8	10
$f(x)$:	0	1	5	21	27

(f) Evaluate

$$\int_{0.5}^{0.7} \sqrt{x} e^{-x} dx$$

by Simpson's $\frac{1}{3}$rd rule dividing the interval into four equal parts.

(g) Calculate the approximate value of

$$\int_{-3}^{3} x^4 dx$$

using trapezoidal rule dividing the interval into six equal parts. Compare with the exact value.

(h) If $\frac{dy}{dx} = x^2 + y^2$ with $y(0) = 0$, find $y(0\cdot 3)$ using Euler's method with $h = 0\cdot 1$.

4. Answer any *one* of the following questions : 10

(a) Let $y = f(x)$ be a function of the independent variable x and $y_0, y_1, \ldots, y_n$ be the values of y corresponding to $x_0, x_1, \ldots, x_n$. Derive Lagrange's interpolation formula to find the value of y at any point x between x_0 and x_n. Use it to find $f(4)$ from the following table : 5+5=10

x	:	0	1	2	5
$f(x)$	:	2	5	7	8

(b) Obtain the piecewise quadratic interpolating polynomial for the function $f(x)$ defined by the following data :

x	:	−2	−1	1	2	4
$f(x)$	:	−29	−8	−2	−5	7

(c) Evaluate $\int_1^2 e^x dx$ using trapezoidal rule considering 2, 3, 5, 9 nodes and Romberg integration method.

(*Continued*)

(d) Solve the initial value problem

$$\frac{dy}{dx} = -2xy^2, \quad y(0) = 1$$

using the mid-point method with $h = 0\cdot 2$ over the interval [0, 1].

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