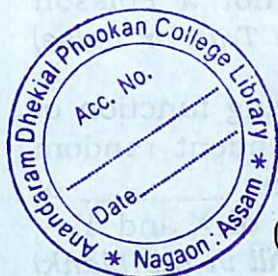


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3 (Sem-5/CBCS) STA HC1



2024

STATISTICS

(Honours Core)

Paper : STA-HC-5016

(Stochastic Processes and Queuing Theory)

Full Marks : 60

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Answer the following questions : $1 \times 7 = 7$
- (a) What is the Markovian property of a stochastic process ?
 - (b) What is absorbing barrier ?
 - (c) The mean of the random variable X in terms of the probability generating function $p(s)$ of X is given by $E(X) = p'(s)$. (State True or False)

Contd.

(d) The difference of two independent Poisson processes is not a Poisson process. (State True or False)

(e) The probability generating function of the sum of two independent random variables X and Y is the _____ of the generating function of X and Y . (Fill in the blank)

(f) Define stochastic process.

(g) What is meant by steady state distribution ?

2. Answer the following questions : $2 \times 4 = 8$

(a) Mention two uses of stochastic process.

(b) Classify the Markov chain with transition probability.

$$\begin{bmatrix} 0 & \frac{1}{3} & \frac{2}{3} \\ \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} & 0 \end{bmatrix}$$

(c) Is poisson process a stationary process ?

(d) Define bivariate probability generating function of a pair of random variable X and Y .

3. Answer **any three** of the following questions : $5 \times 3 = 15$

(a) Prove that the sum of two independent Poisson process is a Poisson process.

(b) Prove that the interval between two successive occurrences of a Poisson process having parameter λ has a negative exponential distribution with mean $\frac{1}{\lambda}$.

(c) Write a note on Order of Markov chain.



- (d) The transition probability matrix of a Markov chain with three states 0, 1, 2 is

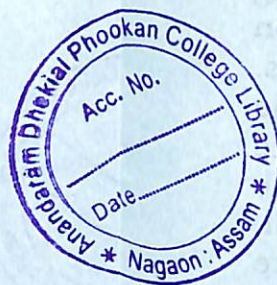
$$P = \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$$

Draw the transition probability graph and hence verify whether the process is transient or not.

- (e) Derive the probability distribution of number of customer in M/M/1 queuing model with finite system capacity.

4. Answer **either** (a) **or** (b) :

- (a) (i) Write a note on "graphical representation of Markov chain." 5
(ii) Derive Chapman-Kolmogorov equation. 5



- (b) (i) Define Poisson process ? Describe the postulates for Poisson process. 5

- (ii) Prove that for a Poisson process $N(t)$, as $t \rightarrow \infty$

$$\Pr \left\{ \left| \frac{N(t)}{t} - \lambda \right| \geq \varepsilon \right\} \rightarrow 0$$

where $\varepsilon > 0$ is a preassigned number. 5

5. Answer **either** (a) **or** (b) :

- (a) (i) Define Queuing system. What is "steady state" of a Queuing system ? 5

- (ii) If $\{N(t)\}$ is a Poisson process and $s < t$, then prove that

$$\Pr \{N(s) = k, N(t) = n\} = \binom{n}{k} \left(\frac{s}{t}\right)^k \left(1 - \frac{s}{t}\right)^{n-k}$$

- (b) (i) Write a note on "Generalization of Independent Bernoulli trials". 5

- (ii) Write a note on "specification of stochastic processes." 5

6. Answer **either** (a) **or** (b) :

(a) Under the postulates for Poisson process, prove that $N(t)$ follows Poisson distribution with mean λt . i.e. $p_n(t)$ is given by the Poisson law.

$$p_n(t) = \frac{e^{-\lambda t} (\lambda t)^n}{n!}, n = 0, 1, 2, \dots$$

10

(b) Cars and bikes arrive at a shopping mall according to Poisson distribution at the rate of 2 cars every 5 minutes. The parking space in front of the shopping mall can accommodate atmost 10 cars/ bikes including the one being served. If the parking lot is full, the cars/bikes are turned away. There is only one service counter. The service time per customer is exponential with a mean of 1.5 minutes. Find

- (i) The Queing model.
(ii) The arrival rate and service rate.

- (iii) The traffic intensity and give interpretation.
(iv) The probability that there are no customer in the shopping mall.
(v) The probability that an arriving customer is turned away.

$$2+2+2+2+2=10$$

