

3 (Sem-1) MAT M 2

2022

MATHEMATICS

(Major)

Paper : 1.2

(Calculus)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

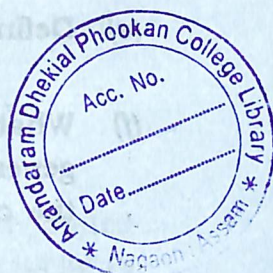
1. Answer the following questions : $1 \times 10 = 10$

(a) Write n th derivative of $\log(ax + b)$.

(b) If $z = f(y/x)$, what is the value of

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y}$$

(c) Write the formula for radius of curvature of a Cartesian curve.



(2)

(d) Two curves $y = f(x)$ and $y' = g(x)$ intersect at the point (x_1, y_1) . Find the condition that they cut orthogonally.

(e) Define double point of a curve.

(f) What is the volume of the solid generated due to the revolution of the circle $x^2 + y^2 = 4$ about X-axis?

(g) Write subnormal to the curve $y^2 = 4ax$ at any point (x, y) .

(h) Write the value of $\int_0^{\pi/2} \cos^7 x \, dx$.

(i) Write the value of

$$\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} \, dx$$

(j) Write the maximum number of asymptotes of algebraic curve of n th degree.

(3)

2. Solve the following questions :

2×5=10

(a) If $y = e^{ax} \sin bx$, show that

$$y_2 - 2ay_1 + (a^2 + b^2)y = 0$$

(b) If $x = r \cos \theta$, $y = r \sin \theta$, prove that

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0, \quad x \neq 0, \quad y \neq 0$$

(c) Find the area of the region bounded by the parabola $y^2 = 4x$ and its latus rectum.

(d) Find the value of $\int_0^{\pi} x \cos^4 x \, dx$.

(e) Find the equation of tangent to the curve $y = be^{-x/a}$ at the point, where it crosses the axis of y .

3. Answer the following questions :

5×2=10

(a) If $y = \sin^{-1} x$, find $(y_n)_0$ where n is odd.

(4)

- (b) Obtain a reduction formula for $\int \sec^n x dx$.

4. Answer either part (a) or part (b) : 10

- (a) (i) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

- (ii) Find the points of inflexion of the curve $y(a^2 + x^2) = x^3$. 5+5=10

- (b) (i) If $u = f(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$, show that

$$\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial y} \right)^2 = \left(\frac{\partial u}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial u}{\partial \theta} \right)^2$$

- (ii) If the normal to the curve $x^{2/3} + y^{2/3} = a^{2/3}$ makes an angle ϕ with x -axis, show that its equation is

$$y \cos \phi - x \sin \phi = a \cos 2\phi.$$

5+5=10

A9/390

(Continued)

(5)

5. Answer the following questions : 5×2=10

- (a) Evaluate :

$$\int_0^{\pi/2} \log \sin x dx$$

- (b) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

6. Answer part (a) or part (b) : 10

- (a) (i) Obtaining n th derivative of x^{2n} , prove that

$$1 + \frac{n^2}{1^2} + \frac{n^2(n-1)^2}{1^2 \cdot 2^2} + \frac{n^2(n-1)^2(n-2)^2}{1^2 \cdot 2^2 \cdot 3^2} + \dots = \frac{(2n)}{(n)^2}$$

- (ii) Show that the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ is

$$\frac{3}{8} \pi a^2.$$

5+5=10

- (b) (i) Find the asymptotes of the curve

$$x^3 + y^3 - 3axy = 0$$

- (ii) Trace the curve $r = a(1 + \cos \theta)$.

5+5=10

A9/390

(Turn Over)

(6)

7. Answer any two questions : $5 \times 2 = 10$

(a) Evaluate :

$$\int_0^{\pi/2} \frac{dx}{5+3\cos x}$$

(b) If $I_n = \int (a^2 + x^2)^{n/2} dx$, show that

$$I_n = \frac{x(a^2 + x^2)^{n/2}}{n+1} + \frac{na^2}{n+1} I_{n-2}$$

(c) Evaluate :

$$\int \frac{dx}{(x^2 - 2x + 1)\sqrt{x^2 - 2x + 3}}$$

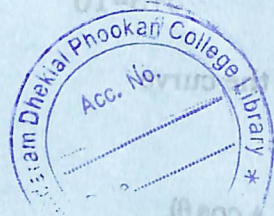
8. Answer the following questions : $5 \times 2 = 10$

(a) Integrate :

$$\int \frac{e^x dx}{e^x - 3e^{-x} + 2}$$

Or

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}}$$



A9/390

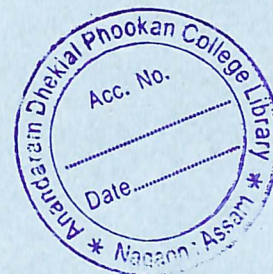
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(7)

(b) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $y = 2x$.

Or

Find the surface area of the solid generated by revolving the cardioid $r = a(1 + \cos \theta)$ about the initial line.



A9—5500/390

3 (Sem-1) MAT M 2