

Total number of printed pages-7

3 (Sem-1 /CBCS) PHY HC 1

2021

(Held in 2022)

PHYSICS

(Honours)

Paper : PHY-HC- 1016

(Mathematical Physics -I)

Full Marks : 60

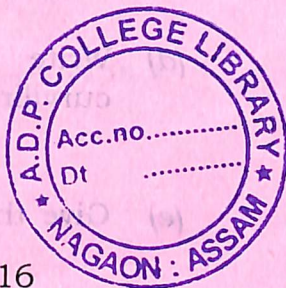
Time : Three hours

***The figures in the margin indicate
full marks for the questions.***

1. Answer the following questions : $1 \times 7 = 7$

(a) State the vector field with respect to Cartesian co-ordinate. Give *one* example.

(b) Show that $\vec{\nabla} \cdot \vec{r} = 3$, where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$.



Contd.

- (c) Write the order and degree of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

- (d) Write the volume element in curvilinear co-ordinate.

- (e) Give the value of $\int_{-\alpha}^{+\alpha} \delta(x) dx$

- (f) Define variance in statistics.

- (g) State the principle of least square fit.

2. Answer of the following questions :

$$2 \times 4 = 8$$

- (a) Find a unit vector perpendicular to the surface, $x^2 + y^2 - z^2 = 11$ at the point (4, 2, 3).

- (b) If $\vec{A} = \vec{A}(t)$, then show that

$$\frac{d}{dt} \left[\vec{A} \cdot \left(\frac{d\vec{A}}{dt} \times \frac{d^2\vec{A}}{dt^2} \right) \right] = A \cdot \left[\frac{d\vec{A}}{dt} \times \frac{d^3\vec{A}}{dt^3} \right]$$

- (c) If $\vec{A}$ and $\vec{B}$ are each irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

- (d) Evaluate $\iint_S \vec{r} \times \hat{n} dS$, where S is a closed surface.

3. Answer **any three** of the following questions :

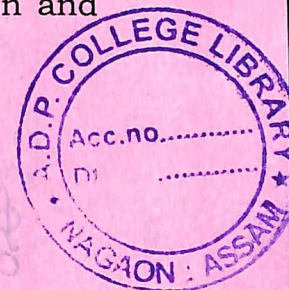
$$5 \times 3 = 15$$

- (a) Prove

$$\iiint_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi) dV = \iint_S (\phi \vec{\nabla} \psi - \psi \vec{\nabla} \phi) \cdot d\vec{S}$$

- (b) Find the integrating factor (IF) of the following differential equation and solve it.

$$(1+x^2) \frac{dy}{dx} + 2xy = \cos x$$



(c) Express curl $\vec{A} = \vec{\nabla} \times \vec{A}$ in cylindrical co-ordinate.

(d) What is Dirac-delta function ? Show that the function

$$\delta(x) = \lim_{\varepsilon \rightarrow 0} \frac{\sin(2\pi\varepsilon x)}{\pi\varepsilon}$$

is a Dirac delta function.

(e) If $\phi(x, y, z) = 3x^2y - y^3x^2$ be any scalar function ϕ , find out

(i) grad ϕ at point (1, 2, 2)

(ii) unit vector $\hat{e}$ perpendicular to surface.

4. Answer **any three** of the following questions : $10 \times 3 = 30$

(a) (i) If $F_1(x, y), F_2(x, y)$ are two continuous functions having continuous partial derivatives

$\frac{\partial F_1}{\partial y}$ and $\frac{\partial F_2}{\partial x}$ over a region R

bounded by simple closed curve C in the x - y plane, then show that

$$\oint_C (F_1 dx + F_2 dy) = \iint_R \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx dy$$

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(ii) A function $f(x)$ is defined

$$\text{as } \begin{cases} 0, & x < 2 \\ \frac{1}{18}(2x+3), & 2 \leq x \leq 4 \\ 0, & x > 4 \end{cases}$$

Show that it is a probability density function. 3

(b) Solve the following differential equations : $5+5=10$

(i) $9 \frac{d^2 y}{dx^2} + 12 \frac{dy}{dx} + 4y = 6e^{-2x/3}$

(ii) $2 \frac{\partial^2 z}{\partial x^2} + 5 \frac{\partial^2 z}{\partial x \partial y} + 2 \frac{\partial^2 z}{\partial y^2} = 0$

(c) (i) A rigid body rotates about an axis passing through the origin with angular velocity $\vec{\omega}$ and with linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$, then prove that,

$$\vec{\omega} = \frac{1}{2} (\vec{\nabla} \times \vec{v})$$

where, $\vec{\omega} = \hat{i}\omega_1 + \hat{j}\omega_2 + \hat{k}\omega_3$

$$\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

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- (ii) If $y = f(x+at) + g(x-at)$, show that it satisfies the equation

$$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

where f and g are assumed to be at least twice differentiable and a is any constant. 5

- (d) (i) Apply Green's theorem in plane to evaluate the integral

$\oint_C [(xy - x^2) dx + x^2 y dy]$ over the triangle bounded by the line $y=0$, $x=1$ and $y=x$. 6

- (ii) Prove that

$$\int_{-\alpha}^{+\alpha} f(x) \delta(x-c) dx = f(c)$$

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- (e) (i) Applying Gauss' theorem, evaluate

$$\iiint_S x dy dz + y dz dx + z dx dy, \text{ where}$$

S is the sphere of radius

$$x^2 + y^2 + z^2 = 1$$

5

- (ii) Evaluate $\nabla^2 \psi$ in spherical co-ordinate. 5

