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A.D.P. College.

3 (Sem-2) MAT M 2

2012

MATHEMATICS

(Major)

Paper : 2.2

(Differential Equation)

Full Marks : 80

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. Answer the following :

1×10=10

- (a) Determine the order, degree, unknown functions and independent variable of the differential equation

$$5\left(\frac{d^4b}{dp^4}\right)^5 + 7\left(\frac{db}{dp}\right)^{10} + b^7 - b^5 = p$$

- (b) Which of the following functions is solution of the differential equation

$$y_2 - y = 0?$$

(i) $y = e^x$

(ii) $y = \sin x$

(iii) $y = 4e^{-x}$

(iv) $y = \tan x$

- (c) When the first-order and first-degree differential equation

$$M dx + N dy = 0$$

(M and N are functions of x and y) is said to be exact?

- (d) Write down the general solution of the differential equation

$$y = px + \sqrt{a^2 p^2 + b^2}, \quad p = \frac{dy}{dx}$$

- (e) Find an integral belonging to complementary function of the differential equation

$$x^2 y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$$

- (f) Write down the necessary condition for integrability of single differential equation

$$Pdx + Qdy + Rdz = 0$$

- (g) Choose the key giving the correct answer :

The partial differential equations can be formed by the elimination of

- (i) arbitrary constants only
- (ii) arbitrary functions only
- (iii) arbitrary functions or arbitrary constants
- (iv) None of the above

- (h) Write down the particular integral of the differential equation

$$y_2 + 3y_1 + 2y = e^x$$

- (i) Give the geometrical interpretation of the differential equation

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

(P, Q and R are functions of x, y and z)

- (j) Construct the partial differential equation by eliminating a and b from

$$z = ax + (1-a)y + b$$

2. Answer the following :

2×5=10

- (a) Determine c_1 and c_2 so that

$$y(x) = c_1 e^{2x} + c_2 e^x + 2 \sin x$$

will satisfy the condition $y(0) = 0$ and $y'(0) = 1$.

- (b) Solve :

$$(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$$

- (c) Solve :

$$\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$$

- (d) Solve :

$$\frac{dx}{dt} = x^2 - 2x + 2$$

- (e) Find the differential equation of the family of curves $y = Ae^{3x} + Be^{5x}$ for different values of A and B .

3. Answer any four parts :

5×4=20

- (a) Find the orthogonal trajectories of the family of curves

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + k} = 1$$

k being the parameter.

- (b) Solve by the method of variation of parameter :

$$y_2 - y = \frac{2}{1 + e^x}$$

- (c) Solve

$$\left(\frac{dy}{dx} \right) (x^2 y^3 + xy) = 1$$

given $y = 0$ when $x = 1$.

- (d) Find a partial differential equation by eliminating the arbitrary function ϕ from

$$\phi(x + y + z, x^2 + y^2 - z^2) = 0$$

- (e) Solve the differential equation

$$\sin^2 x \left(\frac{d^2 y}{dx^2} \right) = 2y$$

given that $y = \cot x$ is a solution.

- (f) Show that the differential equation of all cones which have their vertices at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

4. Answer either (a) and (b) or (c) and (d) : 5+5

(a) Solve :

$$(D^2 + 3D + 2)y = e^{2x} \sin x$$

(b) Solve :

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10 \left(x + \frac{1}{x} \right)$$

(c) Solve :

$$p^2 + 2py \cot x = y^2$$

(d) Show that the following equation is exact and hence solve it :

$$\left\{ y \left(1 + \frac{1}{x} \right) + \cos y \right\} dx + \{ x + \log x - x \sin y \} dy = 0$$

5. Answer either (a) or (b) and (c) : 5+5

(a) Reduce the differential equation

$$\frac{d^2 y}{dx^2} + P(x) \frac{dy}{dx} + Q(x)y = R(x)$$

$$\text{to the form } \frac{d^2 v}{dx^2} + Iv = S$$

(I, S are functions of x) to solve the differential equation. Hence solve the following equation :

$$y_2 - 4xy_1 + (4x^2 - 3)y = e^{x^2}$$

- (b) If $y = y_1(x)$ and $y = y_2(x)$ are two solutions of the equation

$$\frac{d^2y}{dx^2} + P(x)\frac{dy}{dx} + Q(x)y = 0$$

prove that

$$y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = c e^{-\int P dx}$$

c being an arbitrary constant.

- (c) Solve the differential equation

$$x^6 y'' + 3x^5 y' + a^2 y = \frac{1}{x}$$

by changing the independent variable x to z .

6. Answer either (a) and (b) or (c) and (d) : 5+5

- (a) Find $f(y)$ such that the total differential equation

$$\left(\frac{yz+z}{x} \right) dx - z dy + f(y) dz = 0$$

is integrable. Hence solve it.

- (b) Solve :

$$xz^3 dx - z dy + 2y dz = 0$$

(c) Solve :

$$\frac{dx}{dt} + 5x + y = e^t$$

$$\frac{dy}{dt} - x + 3y = e^{2t}$$

(d) Solve the simultaneous equations :

$$\frac{adx}{yz(b-c)} = \frac{bdy}{zx(c-a)} = \frac{cdz}{xy(a-b)}$$

7. Answer either (a) and (b) or (c) and (d) : 5+5

(a) Solve by Lagrange's method :

$$p + q = x + y + z$$

(b) Find the integral surface of the partial differential equation

$$(x-y)p + (y-x-z)q = z$$

through the circle $z=1, x^2 + y^2 = 1$.

(c) Solve by Charpit's method :

$$(p^2 + q^2)y = qz$$

(d) Find the complete integral of

$$z^2(p^2 z^2 + q^2) = 1$$

Find also the singular integral if it exists.

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