

2 0 1 2

PHYSICS

(Major)

Paper : 2.1

Full Marks : 60

Time : 2½ hours

*The figures in the margin indicate full marks
for the questions*

GROUP—A

(Mathematical Methods)

(Marks : 35)

1. Answer the following questions : 1×4=4

(a) What are coordinate curves in curvilinear coordinate system?

(b) What is the value of $\oint_C \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) d\vec{l}$ //

(c) Define scale factors in curvilinear coordinates.

(d) Write the value of $\Gamma(1)$.

2. Answer the following questions :

2×3=6

(a) Evaluate $\int \vec{A} \times \frac{d^2 \vec{A}}{dt^2} dt$.

(b) Prove that $\oint_C (Mdx + Ndy) = 0$ around every closed curve C in a simply connected region iff $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ everywhere in the region.

(c) For an orthogonal curvilinear coordinate system, show that $\hat{e}_1 = h_2 h_3 \vec{\nabla} u_2 \times \vec{\nabla} u_3$, where the symbols stand for usual meanings.

3. Find the square of the element of arc length in cylindrical coordinates and hence find the corresponding scale factors.

3+2=5

Or

A fluid of density $\rho(x, y, z, t)$ moves with velocity $\vec{v}(x, y, z, t)$. If there are no sources and sinks, then use divergence theorem to show $\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$, where $\vec{J} = \rho \vec{v}$.

5

4. Answer either (a) or (b) and (c) :

Either

(a) State and prove Stokes' theorem. 2+8=10

Or

(b) If

$$\vec{A} = (2x^2 + 3y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$$

evaluate $\int_C \vec{A} \cdot d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the path $C: x = t, y = t^2, z = t^3$. 5(c) If $\vec{F} = 2xz\hat{i} - x\hat{j} + y^2\hat{k}$, evaluate $\iiint_V \vec{F} \cdot dV$,where V is the region bounded by the surface $x = 0, y = 0, y = 6, z = x^2, z = 4$. 5

5. Answer either (a) and (b) or (c), (d) and (e) :

Either

(a) Express $\vec{\nabla} \times \vec{A}$ in orthogonal curvilinear coordinates. 7(b) Show that $\int \delta(a-x) \delta(x-b) dx = \delta(a-b)$. 3

Or

(c) Express $\vec{\nabla} \phi$ in cylindrical coordinates. 4

(d) Evaluate : 1

$$\int_{-\infty}^{+\infty} (ax^2 + bx + c) \log \frac{x}{2} \delta(x-2) dx$$

(e) From the definition of gamma function, establish that $\Gamma(n+1) = n\Gamma(n)$. Hence find $\Gamma(\frac{7}{2})$ provided $\Gamma(\frac{1}{2}) = \frac{\pi}{2}$. 3+2=5.

GROUP—B

(Properties of Matter)

(Marks : 25)

6. Answer the following questions : 1×3=3

- (a) What is the range of values of Poisson's ratio σ in elasticity?
- (b) What is the angle of contact for pure water and clean glass?
- (c) What is the dimension of coefficient of viscosity?

7. The pressure inside a soap bubble of radius 1 cm can balance a 1.4 mm column of oil of specific gravity 0.80. Calculate the surface tension of the soap solution ($g = 10 \text{ m/s}^2$). 2

8. Answer any two of the following questions : 5×2=10

- (a) Show that a shear θ is equivalent to half a tensile strain $\theta/2$ and half a compressive strain $\theta/2$ at right angles to each other.
- (b) Find the expression for the relation between surface tension and surface energy as $E = S - T \frac{dS}{dT}$, where E , S , T are surface energy, surface tension and absolute temperature respectively.

- (c) Consider a fluid having coefficient of viscosity η and density ρ flowing through a narrow cylindrical tube of radius r and length l . If P be the pressure difference in the liquid at the two ends, show that the volume of the fluid flowing in time t is

$$V = \frac{\pi P r^4}{8 \eta l} t$$

9. Answer either (a) and (b) or (c) and (d) of the following questions :

Either

- (a) Consider a cylindrical rod of length l and radius r , of material of coefficient of rigidity n . If the cylinder is clamped at one end and twisted at the other through an angle θ , then find an expression for twisting couple per unit twist of the cylinder. 7

- (b) Show that the isothermal bulk modulus of a gas is equal to its pressure. 3

Or

- (c) Find an expression for depression of the free end of a cantilever which is loaded with W and also consider its own weight W_0 as effective. Hence find the expression if it bends under own weight only. 6+1=7

(6)

- (d) Calculate the amount of work done in stretching by 1 mm a piece of steel wire 1 mm^2 in cross-section and 1 m in length. Young's modulus of steel is $2 \times 10^{11} \text{ N/m}^2$.

3

★ ★ ★